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Strength and Static Fatigue of Abraded Glass Under Controlled Ambient Conditions: II, Effect of Various Abrasions and the Universal Fatigue Curve

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A controlled grit blast was found to be a reproducible method of producing standardized damage to a glass surface. The effects of grit size, blast pressure, and amount of grit on the strength of the resulting specimens are reported. Static fatigue curves (strength vs. load duration) were obtained for specimens immersed in room-temperature distilled water and in liquid nitrogen (77°K.) after the specimens had been subjected to various abrasion treatments. The low-temperature strength was independent of load duration, and for surface damage of simple geometry it was inversely proportional to the square root of the initial crack depth, consistent with the Griffith theory. Abrasions of different geometry produced differing static fatigue curves at room temperature, and in one case curves actually crossed. If, however, the strength values for each abrasion were divided by the low-temperature strength for that abrasion and plotted vs. a reduced time coordinate, all the data could be fitted to a single "universal fatigue curve." This analysis led to a clear distinction between "linear" and "point" flaws, the former being flaws (such as emery scratches) which have an extension in a direction perpendicular to the applied stress and the latter being of a more localized character. Linear flaws fatigue more rapidly than point flaws by a factor of fifty and for each type of damage the fatigue rate is inversely proportional to the exponential of the initial flaw depth. A detailed analysis of the data in terms of several static fatigue theories from the literature shows that none of them provides a complete and adequate explanation of these results.

I. Introduction

SINCE the work of Griffith[1] in 1920, it has come to be generally recognized that the failure of bulk glass to exhibit the very high strengths (of the order of 1,000,000 to 5,000,000 lb. per sq. in.) predicted by various theoretical approaches can be attributed to cracks or flaws in the specimens. The possible natural occurrence of such cracks resulting from the network structure of the material has been the subject of considerable discussion in the literature dealing with the strength of glass. It has been clearly demonstrated by now, however (see Part I of this series[2]), that bulk glass (as distinguished from fibers) can, with proper precautions, be prepared with a strength of the order of 100,000 to 200,000 lb. per sq. in. Using the Griffith formula,* the largest cracks which can be present in such specimens are determined to be about 0.07μ in dimension.

To account for the usual technical strength of glass (of the order of 10,000 lb. per sq. in.), it is necessary that cracks of about 7 μ (3×10^{-4} in.) in depth be present. In view of the fact that bulk glass can be prepared in which the largest possible crack is only $^1/_{100}$ of this size, it is apparent that these lower strengths must be attributable to some type of flaw or damage which is not inherent in the glassy structure itself.

Experimentally it is found by examination of the fracture markings that in good-quality glass, with no observable volume defects, breakage at low or moderate stresses always begins at a surface. In addition, the effect of etching on strengthening glass specimens and the readily demonstrable weakening effects of any mechanical contact with high-strength specimens points conclusively to surface imperfections as the source of nearly all moderate or low strength values. Such imperfections are generally the results of mechanical damage to the surface. Minute bruises and scratches are readily formed when a glass surface touches a hard object. Even when such damage is not visible microscopically a consideration of the history of any specimen whose strength is less than, for example, 100,000 lb. per sq. in. will invariably show that there has been some surface contact during the history of the specimen.

Any study of the strength of glass, except for pristine high-strength specimens, thus can be considered as a study of the growth under stress, either slow or catastrophic, of pre-existing surface cracks. An understanding of the role of such surface damage is a prerequisite to understanding the strength of glass, and its control or elimination is a key to improved technical strength of glass articles.

There are four important aspects to the behavior of damage to a glass surface. These can be characterized conveniently as (1) flaw geometry, (2) flaw growth, (3) flaw production, and (4) flaw statistics.

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[1] (*a*) A. A. Griffith, "Phenomena of Rupture and Flow in Solids," *Trans. Roy. Soc.* (*London*), A221, 163–98 (1920); abstracted in *J. Am. Ceram. Soc.*, 4 [6] 513 (1921).
(*b*) A. A. Griffith, "Theory of Rupture," *Proc. First Intern. Congr. Appl. Mechanics, Delft*, pp. 55–63 (1924).

[2] R. E. Mould and R. D. Southwick, "Strength and Static Fatigue of Abraded Glass Under Controlled Ambient Conditions: I, General Concepts and Apparatus," *J. Am. Ceram. Soc.*, 42 [11] 542–47 (1959).

* $\sigma \approx \sqrt{E\alpha/c}$; assume that Young's modulus $E = 10^7$ lb. per sq. in., surface tension $\alpha = 0.0031$ lb. per in., strength $\sigma = 100{,}000$ lb. per sq. in.; then $c \approx \times 10^{-6}$ in. or 0.07 μ.

(1) *Flaw Geometry*

In its simplest form the problem of flaw geometry can be stated as follows: Given a surface flaw or crack of a known size and shape, what macroscopic tensile stress in the material will produce catastrophic growth and failure? For cracks of simple shape this problem can be treated mathematically either in terms of surface energy and strain energy[1, 3] or in terms of stress-concentration factors and the ultimate strength of the material.[4] Orowan[5] has shown the approximate equivalence of the two approaches.

In contrast to the ideal cracks which have been treated mathematically, the geometry of actual damage to a glass surface is usually not simple, and it might appear to be a hopeless task to attempt to relate the strength associated with a given type of surface treatment to the shape of the flaws produced by that treatment. Studies of the grinding and polishing of glass[6, 7] have shown, however, that, in general, damage to a glass surface produces cracks which extend to an appreciable depth below the apparent depth of the damage. It seems likely that such cracks have a much simpler shape than the surface manifestations of the damage and it is possible that they can be related to the simple cracks mentioned above.

(2) *Flaw Growth*

This is essentially the problem of static fatigue or of the delayed failure of glass under stress. This phenomenon is believed to be related to the growth of surface cracks in the material and the resulting change in the stress-concentration factor associated with those cracks. A complete understanding would involve a knowledge of the dependence of crack growth on applied stress, ambient conditions, and previous history of the specimen. The flaw geometry of the initial crack and the way in which it changes with crack growth should also play a significant role.*

(3) *Flaw Production*

What size and shape of flaw is produced by a given treatment of the glass surface and (by (1) and (2) above) what are the resulting strength values? This problem has been studied with respect to diamond scratches by Holland and Turner[8] and more recently by Ord[9] and by Levengood[10] and for abrasion with a hemispherical abrader by Ghering and Turnbull.[11] Because of the great variety of types and degrees of abuse to which glass may conceivably be subjected, a complete answer to the question is not feasible. It is possible, however, that certain common types of abrasions can be categorized and the resulting strength related to the type and degree of damage inflicted.

In the absence of flaws or cracks produced by mechanical action, the question remains, as mentioned above, as to the possible occurrence of inherent flaws related to the nature of the glassy network. As stated above, strength evidence for bulk glass now limits the possible size of such flaws to about 0.1μ and evidence on fibers suggests that the actual limit may be much smaller.

(4) *Flaw Statistics*

A specimen in uniform tension fails at the most severe flaw present. The statistics of strength distributions thus fall into the difficult field of extreme value theory.[12] This subject has greatest interest in connection with the so-called "size effect" and with attempts to deduce the frequency and distribution of postulated inherent flaws in the glass structure from the distribution of strength values for high-strength specimens. In the case of artificially or mechanically produced damage to bulk glass, the distribution of flaw sizes is of less general interest since it will be more dependent on the nature of the damaging process than on the structure of the material itself.

Although a complete understanding of the role of flaws and cracks in determining the strength of glass would involve an understanding of all four factors outlined above, the aim of the present paper is somewhat less ambitious. Two subjects are reported: (1) a study of grit blasting as a means of producing controllable, reproducible damage to glass specimens for strength tests and (2) measurements of strength and static fatigue of certain types of artificial damage under controlled conditions. The study of grit blasting relates to its use as a tool in studying factors other than surface damage which affect strength. The measurements on various abrasions are directed to the first three of the four more basic problems outlined. The fourth, flaw statistics, is not considered.

II. Grit-Blast Abrasion

(A) Apparatus and Methods

Using the grit blaster and impact tester described in Part I of this series,[2] a systematic study of the effect of grain size and blast pressure on the strength of grit-blasted specimens was carried out. The specimens were standard soda-lime glass microscope slides approximately 3 by 1 by 0.040 in. The abraded spot, approximately $1/8$ by $1/4$ in. in size, was placed in the center of one face with its short dimension parallel to the long dimension of the specimen. The slides were then tested in cross bending in the impact tester so that the abraded face was in tension. All fractures originated in the grit-blasted spots. Span length was $2\frac{1}{2}$ in., and the impact load was applied to the center of the compression face through a small coil spring on the impact pendulum.

Before grit blasting, the grit was sieved and baked as described in Part I. In addition, the slides were baked at 200°C. and allowed to cool to room temperature along with the grit in desiccators containing Drierite.† Slides were tested dry in normal room atmosphere immediately after abrading, and each value reported represents the average for twenty specimens.

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[3] R. A. Sack, "Extension of Griffith's Theory of Rupture to Three Dimensions," *Proc. Phys. Soc. (London)*, 58, 729–36 (1946).

[4] H. A. Elliott, "Analysis of Conditions for Rupture Due to Griffith Cracks," *Proc. Phys. Soc. (London)*, 59, 208–23 (1947).

[5] E. Orowan, "Energy Criteria of Fracture," *Welding J. (N. Y.)*, 34, 1575–78 (1955).

[6] W. C. Levengood and W. E. Fowler, "Morphology of Fractures in Polished Glass Surfaces," *J. Am. Ceram. Soc.*, 40 [1] 31–34 (1957).

[7] E. Brüche and H. Poppa, "Polishing of Glass," *J. Soc. Glass Technol.*, 40 [197] 513-19T (1956); *Ceram. Abstr.*, 1958, September, p. 231*i*.

* See star footnote following footnote 2 on p. 543 of Part I of this series for comment on the practical significance of static fatigue.

[8] A. J. Holland and W. E. S. Turner, "Effect of Transverse Scratches on Strength of Sheet Glass," *J. Soc. Glass Technol.*, 21 [87] 383–94T (1937); *Ceram. Abstr.*, 17 [9] 302–303 (1938).

[9] P. R. Ord, "Observations on Glass Cutting by Diamond," *J. Soc. Glass Technol.*, 41 [201] 245-58T (1957); *Ceram. Abstr.*, 1959, April, p. 97*h*.

[10] W. C. Levengood, "Effect of Origin Flaw Characteristics on Glass Strength," *J. Appl. Phys.*, 29 [5] 820–26 (1958); *Ceram. Abstr.*, 1958, September, p. 230*i*.

[11] L. G. Ghering and J. C. Turnbull, "Scratching of Glass by Metals," *Bull. Am. Ceram. Soc.*, 19 [8] 290–94 (1940).

[12] Statistical theories of fracture have been summarized by B. Epstein, "Statistical Aspects of Fracture Problems," *J. Appl. Phys.*, 19 [2] 140–47 (1948); *Ceram. Abstr.*, 1949, November, p. 269*d*.

† Subsequent experiments showed that the precaution of baking specimens before abrading produced no discernible effect on the results.

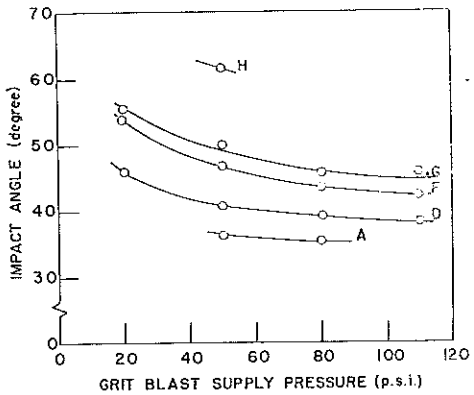

Fig. 1. Impact strength vs. blast pressure for various grit sizes.

Curve	Grit size	Sieve sizes	Amount of grit
A	30	Not sieved	1/4
D	60	50/60	1/4
F	60	80/100	1/2
G	80	80/100	1/2
H	120	100/140	1

Sieve sizes indicate mesh sizes of U. S. Standard sieves between which fraction of grit used was retained. Under "Amount of grit," one unit = approximately 0.05 cc. or 0.10 gm. of grit.

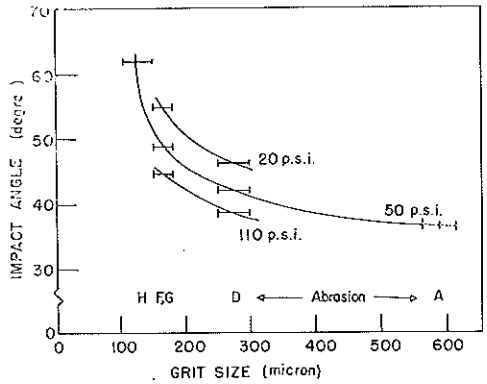

Fig. 2. Impact strength vs. grit size for various blast pressures; grit-blasted microscope slides. Ends of horizontal bars correspond to openings on sieves used. Letters A, D, F, G, and H as in Fig. 1.

(B) Results and Discussion for Grit Blast

The results of these measurements are plotted in Figs. 1 and 2. Strengths are reported in degrees of fall of the impact pendulum to produce breakage since these measurements were performed with an early version of the impact tester for which a complete stress calibration was not obtained. Bending stress is approximately proportional to this quantity, and subsequent measurements showed that 40° corresponded to a bending stress of about 8700 lb. per sq. in.

An examination of the results of these tests yields the following information with respect to the effects of the variables associated with grit blasting on the strength of the resulting specimens:

(1) *Blast Pressure*

As would be expected, strength decreased with increasing blast pressure. The relatively small decrease with increasing pressure above about 50 lb. per sq. in. may indicate that, for the nozzle geometry used, turbulence in the gas flow provided a limiting factor on the severity of the blast. From the standpoint of producing reproducible abrasions it is obvious that the pressure must be controlled and it is preferable to use relatively high pressures where the degree of damage inflicted is relatively insensitive to pressure changes.

(2) *Grit Size*

The strength of the grit-blasted specimens also depended on the grain size of the grit used. Of special interest in this connection are curves (*F*) and (*G*) of Fig. 1. These show that strength differences can result even with sieved grit when the distribution of sizes in the initial grit supply changes. For accurately reproducible strength results it is necessary not only to sieve the grit used but to start with similar samples before sieving. An obvious interpretation of these results is that although the maximum and minimum sizes for the two samples of grit (determined by the sieves used) were the same, the distribution of sizes within those limits differed enough to produce different strength results.

(3) *Amount of Grit*

Experiments were conducted in which the amount of grit used was varied by factors of up to four to one. Over this range the resulting strength values were independent of the amount of grit used. In general, this appeared to be true as long as the abrasion was not so severe as to reduce the thickness of the slide appreciably over the area of the abrasion. Although no significant differences in per cent standard deviations of the strength values were observed with varying amounts of grit, it appeared that best results were obtained when the amount was adjusted so as to give a large number of individual abrasions evenly distributed over the damage area, with each abrasion, in general, surrounded by some of the original glass surface. Photomicrographs of such abrasions are shown later in Fig. 4.

(4) *Variability of Strength Values*

For all the grit-blast abrasions studied the standard deviations of the strength values were between 5 and 10% of the average for each specimen. This is considerably lower than the usual variability of specimens without artificial abrasions (20 to 25%) and somewhat lower than is obtained with hand abrasion using emery cloth (10 to 15%). For comparison, Teague and Blau[13] obtained coefficients of variation of about 11% in measuring the pressure strength of grit-blasted bottles, and Holland and Turner[14] report coefficients ranging from about 6 to 15% for bending tests on laths prepared in a variety of ways. Holland and Turner also obtained coefficients of 6 to 10% for the strength of specimens containing carefully produced diamond scratches.[8]

III. Strength and Static Fatigue With Various Abrasions

In these tests static fatigue curves were measured for soda-lime glass microscope slides each of whose surfaces had been subjected to one of six different abrading treatments. Early in the course of the present studies it became apparent that glass with different kinds of surface damage showed not only different strengths but also different ratios of long- to short-time strengths indicative of different rates of static fatigue.

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[13] J. M. Teague, Jr., and H. H. Blau, "Investigations of Stresses in Glass Bottles Under Internal Hydrostatic Pressure: I, Photoelastic Studies of Fosterite Models; II, Breakage Studies of Glass Bottles Under Internal Hydrostatic Pressure; III, Electric Strain Gauge and Brittle Coating Studies," *J. Am. Ceram. Soc.*, 39 [7] 229–52 (1956).

[14] A. J. Holland and W. E. S. Turner, "Breaking Strength of Glass; Effect of Flaws and Scratches," *J. Soc. Glass Technol.*, 20 [80] 279–302T (1936); *Ceram. Abstr.*, 17 [1] 11 (1938).

It was in an attempt to evaluate this effect and to shed some light on the role of flaw geometry in the determination of strength and static fatigue that these measurements were undertaken.

(A) Experimental Methods

(1) *Specimen Preparation*

The abrasions studied in these tests were produced by two methods, both of which have been used previously in this laboratory and elsewhere to produce glass surfaces of relatively uniform and reproducible strengths. The two methods used were hand abrasion with emery cloth and grit blasting.

The emery cloth abrasions were performed as follows. Each specimen was abraded by dragging a new half-inch square of cloth under fingertip pressure across its surface four times, rotating the square 90° after each stroke. Three grades of emery cloth were used with the abrasions formed at right angles to the principal tensile stress and one of these grades was also used with the abrasions parallel to the stress.

The grit-blast method has already been described. Two combinations of grit size and blast pressure were used to provide two series of tests on grit-blasted slides.

Each test set consisted of twenty slides and, in general, all sets with any given abrasion were prepared at the same time, the slides being abraded and added to each set in rotation. After abrasion, the slides were allowed to age for approximately 24 hours in room atmosphere. Another study* had shown that changes of the strength in the absence of stress, which are encountered immediately after abrading, have become so slow after 24 hours that further changes during the test period can be neglected.

(2) *Static Fatigue Tests*

Since at the time of these tests the effect of various ambient conditions during test had not as yet been studied systematically, all tests were conducted with the specimens wetted with distilled water to provide a common basis for comparison. Each slide was wetted immediately before test. For loads of short duration one dipping was sufficient to wet the slides for the duration of the test. For longer times a small pan was inserted in the tester so that when the slide was deflected its lower surface dipped into the water. Fresh distilled water was used for each set of twenty specimens.

Tests were conducted in the electromagnetic loader and dead-weight tester which have been described in Part I of this series. In these testers the specimens were loaded in cross bending with the abraded surface in tension. The load was applied by a single knife-edge at the middle of a $2^1/_2$-in. span.

All the tests were of the stepwise pulse type in which a constant load is applied for a given time and then removed. If the specimen has not broken, a greater load is applied for the same duration, and so on until breakage occurs. If the loading increments in such a test are within the proper range and one-half the loading increment is subtracted from each value, it can be shown that the results obtained are comparable with those which would be obtained in a true static fatigue test, i.e., one in which a constant load is applied and the time to fracture measured.

As discussed by Baker and Preston,[15] the criterion for this to be true is that the loads before that which breaks the specimen shall have a negligible weakening effect. This can be tested experimentally by varying the increment or step size and determining whether this produces any change in the measured strength. Results of such a test for two load durations are presented in Table I. From the table it can be seen that for the three increments used (corresponding to approximately 5, 10, and 20% of the breaking stress) the resulting strength values agreed very well. There is some evidence that the smaller increments produced slightly lower values, but this variation is not statistically significant and even if real would be negligible in most tests. In the tests reported below an increment of approximately 10% of the breaking stress was used in all tests.

Table I. Effect of Loading Increment on Strength of Abraded Glass in Stepwise Loading*

Loading increment (lb./sq. in.)	Breaking strength (lb./sq. in.)	
	0.012-second pulse	0.44-second pulse
366	8240	6900
713	8320	7030
1426	8340	7190

* Slides grit blasted, stored 24 hours in room atmosphere, and tested wet with distilled water.

Table II. Strength of Specimens in Liquid Nitrogen*

Load duration (second)	Breaking strength (lb./sq. in.)	
	Liquid nitrogen (77°K.)	Water (23°C.)
0.0025	12,510	9860
0.050	12,580	8360
0.82	12,450	7170
15	12,580	5680

* Slides grit blasted, aged overnight in distilled water, and tested immersed in liquid nitrogen or distilled water.

(3) *Liquid Nitrogen Tests*

Some time after the completion of the foregoing static fatigue tests the electromagnetic tester was equipped so that tests could be conducted with the specimens immersed in liquid nitrogen.

Typical results for one abrasion are shown in Table II along with values for tests in water for comparison. It can be seen that the strength in liquid nitrogen is independent of load duration over the range studied within an accuracy of about 1%. Thus under these conditions static fatigue appears to be eliminated and a single strength value obtained. Strength tests therefore were conducted for the six different abrasions of this study with the specimens immersed in liquid nitrogen in order to obtain the initial strength associated with each type of abrasion.

Between the completion of the fatigue tests and the undertaking of the low-temperature tests, aging experiments† showed that water immersion gave a more reproducible aging treatment for fresh abrasions than storage in room atmosphere. Specimens for the low-temperature tests were immersed in distilled water for 24 hours between abrasion and test. Control specimens were abraded along with those for the low-temperature tests and tested wet after this same aging treatment to provide a basis for comparison with the fatigue results.

(4) *Measurement of Depth of Abrasions*

In addition to the strength tests, the emery cloth abrasions were examined with an optical microscope both before and after a mild acid etch. An estimate was made of the depth

* Part III of this series.

[15] T. C. Baker and F. W. Preston, "Fatigue of Glass Under Static Loads," *J. Appl. Phys.*, 17 [3] 170–78 (1946); *Ceram. Abstr.*, 1946, August, p. 149.

† Part III of this series.

Table III. Strength of Glass with Various Abrasions*

Load duration (second)	Grit blast (a) Severe†	Grit blast (b) Mild‡	Emery cloth (c) 150 grit§	Emery cloth (d) 600 grit‖	Emery cloth (e) 320 grit‖	Emery cloth (f) 150 grit‖
0.0025	10,050	9,704 9,710		9,740	8,260	6,750
0.012	9,430	9,180 9,180	13,850 14,060	9,280	7,830	6,280
0.05		8,350 8,590 8,390	13,210 12,970		6,900	
0.09 0.226	8,440	7,730 7,640 7,880	11,390 11,940	7,720	6,150	5,670
0.44 0.82	7,590	7,190 7,080 7,190	10,910 11,670	7,110	5,260	5,100
1.5 4.0	7,010	6,560 6,970 6,500	9,370 9,710	6,370	4,720	4,910
10 15	6,450	6,130 6,010 5,760	8,330 8,590	5,310	4,400	4,370
60	5,680	5,370 5,460	7,450 7,420	4,850 4,500	3,800	3,860 3,360
600		4,790	6,090 6,240	3,800	3,180	2,860

* Specimens abraded with emery cloth or grit blast, stored in room atmosphere for 24 hours, and tested wet with distilled water. Strengths in pounds per square inch.

† No. 60 grit sieved between No. 70 and 80 sieves; one-half unit; blast pressure, 30 lb. per sq. in.

‡ No. 60 grit sieved between No. 50 and 60 sieves; one-quarter unit; blast pressure, 80 lb. per sq. in.

§ Abrasion parallel to tensile stress.

‖ Abrasion perpendicular to tensile stress.

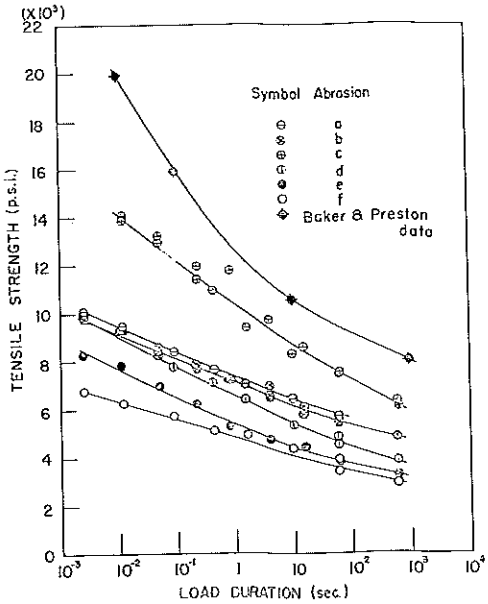

Fig. 3. Static fatigue curves for specimens with various abrasions. Specimens tested in cross bending immersed in distilled water; twenty specimens each point. See Table III for description of abrasions. Data of Baker and Preston, footnote 15, shown for comparison.

of the abrasions by the use of a polishing and etching technique. In this technique a slide which previously had been abraded over its entire surface by one of the methods under study was mounted in a suitable jig and polished with rouge at the edge of a felt wheel so that a very shallow trough was formed in the abraded surface. After a light acid etch to remove the plastically disturbed layer left by the polishing, the slide was examined in a microscope to determine the location of the points at which the depth of the trough equaled the depth of the deepest flaws present. Measurement of the thickness of the slide at these points and comparison with measurements made before polishing then gave an estimate of the depth of the flaw.

(B) Results for Various Abrasions

(1) *Strength Values and Static Fatigue Curves*

The results of the tests are shown in Table III. In this table the tensile breaking stress for each type of abrasion at each load duration is given. For the hand abrasions the standard deviations of the strengths were from 8 to 12% of the average value except in the case of the 150-grit abrasion parallel to the stress, in which case the standard deviations ran as high as 20% of the average. For the grit-blast abrasions the standard deviations were from 5 to 9% of the average value.

In Fig. 3 the results are plotted in the form of static fatigue curves showing strength vs. load duration for each type of abrasion. The data obtained by Baker and Preston[15] are also included for comparison. An examination of the figure shows that all the abrasions studied produced the same general fatigue behavior. Strength was greatest for short-time loads and decreased steadily as the load duration increased. There were differences, however, in the detailed behavior of the different abrasions as evidenced by differences in the slopes of the curves and by the fact that in one case two curves actually cross.

Table IV. Low-Temperature Strength*

Abrasion	Breaking strength (lb./sq. in.) Liquid N_2 (77°K.)	Breaking strength (lb./sq. in.) Distilled water (23°C.)
Grit blast		
30 (70/80)$^{50\ 1/2}$	16,180	11,190
80 (50/60)$^{50\ 1/4}$	13,020	9,530
Emery cloth		
600 grit†	19,540	9,310
320 grit†	13,850	7,840
150 grit†	10,620	6,580
150 grit‡	21,850	12,660

* Specimens abraded with emery cloth or grit blast, aged 24 hours in distilled water, and tested with 0.012-second pulse in liquid N_2 or distilled water. Strengths in pounds per square inch.

† Abrasion perpendicular to stress.

‡ Abrasion parallel to stress.

(2) *Low-Temperature Results*

The results of the measurements in liquid nitrogen are shown in Table IV along with the results for the control sets which were tested wet at room temperature. The lack of close agreement in certain cases between the control sets of Table IV and the 0.012-second values of Table III reflects both the difference in aging treatment and, in the case of the grit blast, a minor modification which was made in the grit blaster in the interval (about 15 months) between the taking of the two sets of data.

(3) *Nature of Abrasions*

Photomicrographs of the various abrasions are shown in Fig. 4. The following points are worth noting:

(1) The grit-blast abrasions ((*a*) and (*b*)) consist of individual spots of damage with an appreciable amount of the

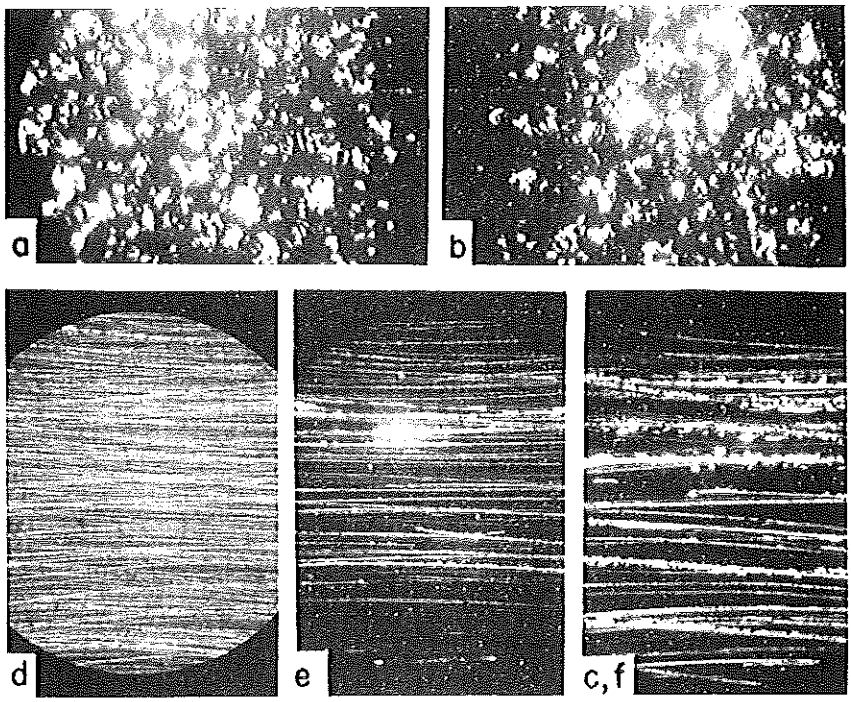

Fig. 4. Photomicrographs of various abrasions. Diameter of field of view is about 0.1 in. See Table III for description of abrasions.

Table V. Estimated Depth of Damage for Various Emery Cloth Abrasions

Abrasion	Depth, c (in. $\times 10^{-4}$)
600-grit emery	2 ± 1
320-grit emery	4 ± 1
150-grit emery	9 ± 1

Table VI. Values of $\sigma\sqrt{c}$ for Emery Cloth Abrasions

Abrasion	$\sigma\sqrt{c}$ (lb./in.$^{3/2}$)
600-grit emery	280
320-grit emery	280
150-grit emery	320

original glass surface intact in the damaged area. The difference in severity between (*a*) and (*b*) is not apparent from the photomicrographs. Under visual microscopic examination the individual damage spots of (*a*) appear somewhat larger and deeper than those of (*b*).

(2) The emery cloth abrasions ((*d*), (*e*), and (*f*)) consist of long scratches with the original surface still present between the individual scratch marks. As the grit size is increased, fewer scratches, each of greater depth and severity, are produced.

The depths of the emery cloth abrasions as estimated by the polish and etch technique are given in Table V. The accuracy of this measurement of depth was approximately ± 0.0001 in. Since the result depended on finding the most severe damage in one particular specimen, however, the values may be in error by a considerably larger amount with respect to being representative of all specimens of a particular type.

(C) Detailed Analysis of Results

(1) *Flaw Depth, Initial Strength, and the Griffith Theory*

It is of interest first to consider whether the strength in the absence of static fatigue (as determined by the tests in liquid nitrogen) shows any relation to the depth of the flaws as measured by the polish and etch technique. According to both the Griffith formula and the theory of stress concentrators, one should expect the product of the breaking stress, σ, and the square root of the flaw depth, c, to be a constant and to be approximately equal to $\sqrt{E\alpha}$ (E = Young's modulus, α = surface energy). As calculated from the data of Tables IV and V, this product for the emery cloth abrasions is $\sigma\sqrt{c} = 300$ (σ in pounds per square inch, c in inches) (see Table VI).

For artificially made cracks Griffith obtained a value of 240 for this product. Griffith made his measurements in room atmosphere at room temperature. The actual agreement is perhaps not as good as would appear from this comparison, since he might have obtained a higher value for measurements at low temperatures where fatigue does not occur. The agreement is, however, reasonably good and indicates that to a first approximation this product is a constant for surface flaws of rather widely differing dimensions.*

If Young's modulus is about 10^7 lb. per sq. in. (as is the case for these specimens), a product $\sigma\sqrt{c} = 300$ corresponds to a surface tension of $\alpha = 0.0090$ lb. per in. or 1600 dynes per cm. This compares with measured values at high temperature of about 300 dynes per cm.[16] This discrepancy may indicate that the surface energy of a freshly *fractured* surface at low

* The constancy of this product should not be confused with that of stress times the square root of the diameter of the mirror area on the fractured surface, as described by Levengood, footnote 10. In general, the mirror is much larger than the original flaw and that product is about 2000 rather than 300.

[16] G. W. Morey, Properties of Glass, 2d ed., p. 199. American Chemical Society Monograph Series No. 124, Reinhold Publishing Corp., New York, 1954. 591 pp.; *Ceram. Abstr.*, **1954**, October, p. 180*h*.

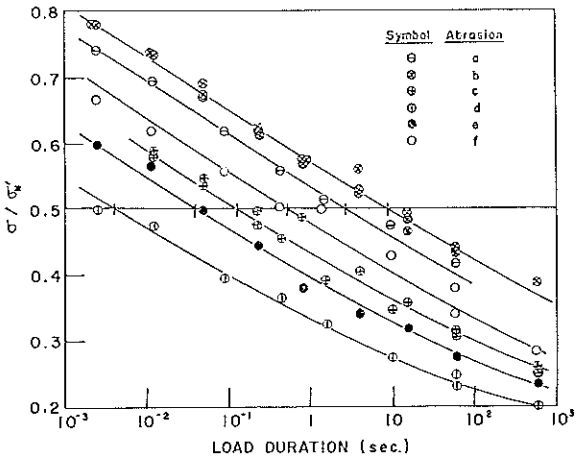

Fig. 5. Reduced strength vs. load duration (log scale) for various abrasions. Strength values, σ, divided by adjusted liquid nitrogen strength (σ_N') for each abrasion. Vertical marks on $\sigma/\sigma_N' = 0.5$ line indicate value of $t_{0.5}$ for each curve. Curves fitting data are appropriate portions of universal fatigue curve (see Fig. 6). See Table III for description of abrasions.

temperature is, as might be expected, higher than that of a freshly *drawn* surface at high temperature.

The value of 1600 dynes per cm. is of course only a rather crude estimate. As shown by various investigators[1, 3, 4] $\sigma = k\sqrt{E\alpha/c}$, where k can have values ranging from 0.81 to 1.27, depending on the geometry of the flaw. This range of k leads to a range in α of 1000 to 2400 dynes per cm. The lower limit of this range is still, however, considerably greater than measured high-temperature values.

(2) Construction of Universal Fatigue Curve

As a means of empirical analysis the data of Table III were calculated on a reduced basis, i.e., with each strength value divided by the corresponding low-temperature strength based on the data of Table IV. As mentioned above, the low-temperature tests reported in Table IV were not exactly comparable with the fatigue tests of Table III because of small differences in the abrading and aging treatments. For this reason an adjusted liquid nitrogen strength σ_N' was used in reducing the fatigue data for each abrasion. This was determined by the relation

$$\frac{\sigma_N'}{\sigma_{0.012}} = \frac{\sigma_N}{\sigma_{\text{control}}} \tag{1}$$

or

$$\sigma_N' = \frac{\sigma_{0.012}}{\sigma_{\text{control}}}\,\sigma_N \tag{1a}$$

σ_N' = adjusted low-temperature strength.
$\sigma_{0.012}$ = 0.012-second wet strength from Table III.
σ_N = liquid nitrogen strength of Table IV.
σ_{control} = 0.012-second wet strength from Table IV.

A comparison of $\sigma_{0.012}$ (Table III) with σ_{control} (Table IV) shows the magnitude of this adjustment for each abrasion. In four cases it is 5% or less, in one (150-grit perpendicular) about 10%, and in one (30 (70/80)^{60} 1/2) about 18%.

The strength values for each abrasion divided by the adjusted low-temperature strength for that abrasion (σ/σ_N') are plotted vs. load duration in Fig. 5. From this figure it can be seen that when the data are plotted in this reduced fashion, the differences in the static fatigue curves of Fig. 3 are largely eliminated, and the results can be represented by a family of very similar curves.

A vertical line crossing each curve of Fig. 5 indicates that load duration for which $\sigma/\sigma_N' = 0.5$. This duration is denoted by $t_{0.5}$. The values of σ_N' and $t_{0.5}$ for each abrasion are given in Table VII.

If the normalized fatigue curves of Fig. 5 are shifted horizontally so that all the characteristic durations ($t = t_{0.5}$) coincide, the plot shown in Fig. 6 is obtained. Since the horizontal axis is plotted on a logarithmic scale, this shift is equivalent to plotting σ/σ_N' vs. $t/t_{0.5}$ (log t' = log t − log $t_{0.5}$; $t' = t/t_{0.5}$).

From Fig. 6 it can be seen that all the experimental points of this study lie reasonably well on one universal fatigue curve when plotted in this reduced fashion. A somewhat better estimate of the degree of fit can be obtained by referring back to Fig. 5, since each of the individual curves of that figure consists of the appropriate portion of the universal curve of Fig. 6.

Based on this analysis, all the results of this study can be expressed by the relation

$$\frac{\sigma}{\sigma_N} = f\left(\frac{t}{t_{0.5}}\right) \tag{2}$$

where σ_N and $t_{0.5}$ for each abrasion are as given in Table VII and f is a single function representing the curve of Fig. 6. It therefore is apparent that the observed differences in the static fatigue curves for the various abrasions do not reflect any basic differences in their over-all fatigue behavior. The differences in the usual fatigue curves of Fig. 3 arise rather because the characteristic time, $t_{0.5}$, differs widely from one abrasion to another and one is, in effect, studying different portions of the same over-all fatigue curve in each case.

Table VII. Values of σ_N and $t_{0.5}$ for Various Abrasions

Abrasion	σ_N' (lb./sq. in.)	$t_{0.5}$ (second)
Grit blast		
30 (70/80)^{60} 1/2	13,620	2.9
80 (50/60)^{60} 1/4	12,450	8.8
Emery cloth		
600 grit*	19,480	0.0043
320 grit*	13,840	0.039
150 grit*	10,140	0.56
150 grit†	23,900	0.14

* Abrasion perpendicular to tensile stress.
† Abrasion parallel to tensile stress.

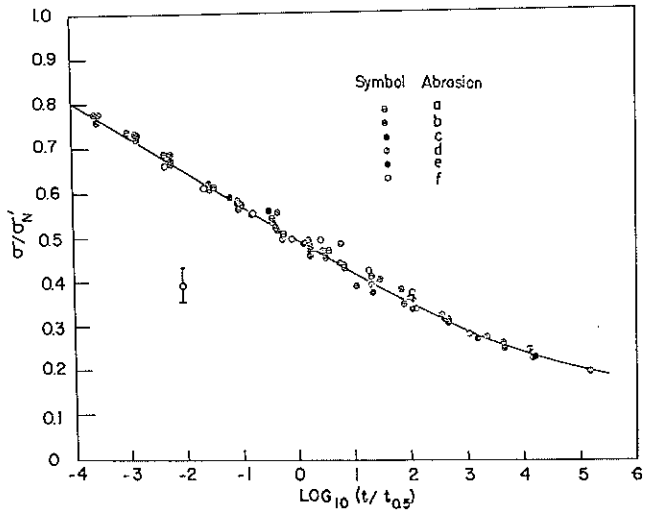

Fig. 6. Universal fatigue curve for various abrasions. Strength divided by liquid nitrogen strength vs. logarithm of load duration divided by $t_{0.5}$ for each abrasion. Vertical bar near upper end of curve indicates approximate uncertainty for individual points. See Table III for description of abrasions.

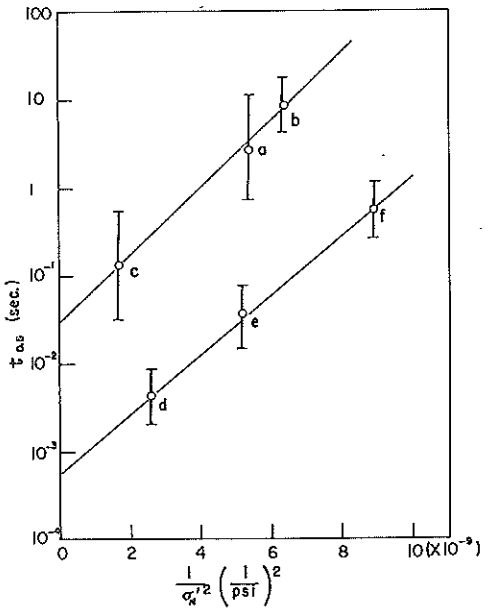

Fig. 7. Characteristic duration ($t_{0.5}$) vs. reciprocal square of liquid nitrogen strength for various abrasions. Error bars indicate estimated uncertainty in $t_{0.5}$ for each case. See Table III for description of abrasions.

It should be noted that the abrasions studied cover only a relatively small range of rather severe damage and correspondingly low strengths. Whether the universal fatigue curve of Fig. 6 will fit high-strength data for pristine massive glass and the still higher values obtained for glass fibers remains to be seen. Bending tests on fibers have shown that they exhibit fatigue properties similar to bulk glass.[17, 18] Unfortunately the low-temperature strength values necessary for an analysis in terms of reduced strength are not available as yet.

(3) Dependence of $t_{0.5}$ on $(1/\sigma_N)^2$

It is of interest to determine whether $t_{0.5}$ as determined above is related to σ_N for each abrasion. If this were the case, a measurement of σ_N, the liquid nitrogen strength, would specify the entire fatigue curve for a glass of a given composition under given test conditions regardless of the type of surface damage present. In Fig. 7 ln $t_{0.5}$ is plotted vs. $(1/\sigma_N)^2$* for the six abrasions. The vertical error bars in this plot are based on the estimated uncertainty in σ_N and indicate the range of $t_{0.5}$ values which would result from such a range in σ_N.

From the plot it appears that the abrasions fall into two distinct groups consistent with their geometry. The emery cloth abrasions perpendicular to the stress form one group and the grit blast and emery cloth parallel to the stress form another. The former are denoted as *linear* flaws since they extend at right angles to the tensile stress and the latter as *point* flaws because of their more localized nature.

Two points are immediately apparent from the figure. First, a specimen with a large initial flaw size, $(1/\sigma_N)^2$ large, and low strength fatigues less rapidly than one with smaller initial flaws and higher strength. The relation between ln $t_{0.5}$ and $1/\sigma_N{}^2$ is approximately linear, although the number of points is insufficient to establish this definitely. Second, the linear flaws fatigue much more rapidly than the point flaws, sustaining a load of 0.5 σ_N for only about $^1/_{50}$ of the time for the point flaws. This suggests either that the linear flaws can grow more readily under the action of water and stress or that perhaps capillary forces permit better access of water to the roots of the linear flaws.†

If the linear relation between ln $t_{0.5}$ and $(1/\sigma_N)^2$ is correct, one can write

$$\ln t_{0.5} = A + \frac{B}{\sigma_N{}^2} \qquad (3)$$

$$t_{0.5} = A_0 e^{B/\sigma_N{}^2} \qquad (3a)$$

where A and B are constants, independent of σ_N, and $A_0 = e^A$.

Equation (2) can then be written as

$$\frac{\sigma}{\sigma_N} = f\left(\frac{t}{A_0 e^{B/\sigma_N{}^2}}\right) \qquad (4)$$

All the data of this study can be fitted by equation (4) if A_0 and B have the following values:

	A_0 (sec.)	B (lb.²/in.⁴)
Point flaws	2.9×10^{-2}	8.7×10^8
Linear flaws	5.5×10^{-4}	7.7×10^8

and f is a function representing the universal fatigue curve of Fig. 6. Numerical values of this function as taken from the figure are tabulated in Appendix I (see p. 592).

It should be emphasized that the foregoing analysis of the data of this study and especially the existence of the universal fatigue curve were established on an empirical basis. More extensive or precise data might show that the reduced static fatigue curves for different abrasions differ significantly and cannot all be fitted by a single curve, especially at extremely long or short durations. However, based on the present results, such deviations if they exist must be quite small (less than about 5%). Even if the universal fatigue curve and equations (2) and (4) should prove to be only a first approximation to some more complicated relation among the strengths of specimens containing various abrasions, the use of $t_{0.5}$ as a parameter characterizing the static fatigue of glass specimens would not necessarily be invalidated since its definition is not dependent on the exact shape of the static fatigue curves. In the same way the relation between $t_{0.5}$ and σ_N as shown in Fig. 7 would also remain unaffected.

(D) Detailed Comparison with Various Theories

Since the discovery of the static fatigue effect for glass, several investigators have attempted to explain the experimental results on the basis of various physical models.[19–24] In this section the experimental results of this study are compared

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[17] O. C. Hansen, Preston Laboratories, Incorporated; unpublished data.

[18] R. E. Mould, "Crossbending Tests of Glass Fibers and the Limiting Strength of Glass," *J. Appl. Phys.*, 29 [8] 1263–64 (1958); *Ceram. Abstr.*, 1959, January, p. 6*d*.

* In Section III (C) (1) this quantity was shown to be (at least approximately) proportional to the initial crack depth.

† The role of water in static fatigue is considered at greater length in Part IV of this series.

[19] J. B. Murgatroyd, "Mechanism of Brittle Rupture in Glass," *J. Soc. Glass Technol.*, 28 [130] 406–31T (1944); *Ceram. Abstr.*, 1946, September, p. 157.

[20] J. L. Glathart and F. W. Preston, "Fatigue Modulus of Glass," *J. Appl. Phys.*, 17 [3] 189–95 (1946); *Ceram. Abstr.*, 1946, August, p. 137.

[21] N. W. Taylor, "Mechanism of Fracture of Glass and Similar Brittle Solids," *J. Appl. Phys.*, 18 [11] 943–55 (1947); *Ceram. Abstr.*, 1948, August, p. 191*g*.

[22] D. A. Stuart and O. L. Anderson, "Dependence of Ultimate Strength of Glass Under Constant Load on Temperature, Ambient Atmosphere, and Time," *J. Am. Ceram. Soc.*, 36 [12] 416–24 (1953).

[23] H. A. Elliott, "Stress Rupture in Glass," *J. Appl. Phys.*, 29, 224–25 (1958).

[24] R. J. Charles, "Static Fatigue of Glass, I-II," *J. Appl. Phys.*, 29 [11] 1549–53, 1554–60 (1958); *Ceram. Abstr.*, 1959, March, p. 76*j*.

Table VIII. Theoretical Expressions for the Static Fatigue Curve*

Equation	Investigator and reference
(*A*) $\log t = -a + \frac{b}{\sigma}$	Murgatroyd, footnote 19 Glathart and Preston, footnote 20 Taylor, footnote 21
(*B*) $\log t \simeq a - \frac{b}{\sigma} - \log \sigma$ (for small t)	Stuart and Anderson, footnote 22
(*C*) $\log t = -a + \frac{b}{\sigma^2}$	Elliott, footnote 23
(*D*) $\log t \simeq -a - b (\log \sigma)$ (for large t)	Charles, footnote 24

* σ = applied stress, t = time until breakage occurs.

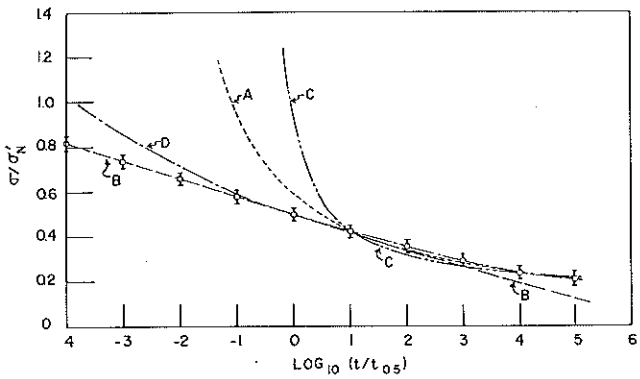

Fig. 8. Comparison of universal fatigue curve with various theories. Points are values of UFC from Fig. 6 with estimated uncertainty as indicated by vertical bars. Curves are best fit based on equations (*A*) through (*D*) of Table VIII.

with the predictions of these theories. It is instructive to note first two points with respect to the present results:

(*a*) If the foregoing analysis of the data for various abrasions in terms of reduced variables is valid, the universal fatigue curve (referred to hereafter as the UFC) of Fig. 6 depicts the time dependence of the strength of glass over nine decades of time. This is an appreciably greater duration than has ever been studied (or is likely to be) for any one group of specimens. Fitting this curve therefore should provide a more stringent test of various analytical expressions than has existed heretofore.

(*b*) Since the UFC has a simple monotonic shape it is likely that each of several analytical expressions with one or two arbitrary parameters will adequately represent the data. For this reason the strong experimental dependence of $t_{0.5}$ on $1/\sigma_N^2$ is likely to provide an even more critical test for a fatigue theory than fitting the reduced curve.

(1) *The Instantaneous Strength*

Items (*a*) and (*b*) provide two criteria by which to judge the adequacy of a theory for the static fatigue of glass. To apply them to the data of this study one needs first to define the concept of the instantaneous strength of a glass specimen. The instantaneous strength (σ_0) is defined as the strength measured at a load duration (or loading rate) such that any decrease in duration (or increase in rate) will produce no further increase in strength.* The possibility of defining σ_0 exists because of the experimental fact that static fatigue is eliminated when tests are conducted under certain conditions. Thus the strength in vacuum as measured by Baker and Preston[15] and the strength at liquid nitrogen temperature as measured in this study and in others[24, 25] satisfy the foregoing definition since they show no dependence on test duration. Experimentally, any departure from low-temperature or vacuum conditions always produces a strength value lower than σ_0.

The existence of an instantaneous strength of course implies that the UFC will become horizontal if extended to small enough values of reduced time. The horizontal asymptote will be σ_0/σ_N, where σ_0 is the instantaneous strength at room temperature and σ_N is the liquid nitrogen strength, i.e., the instantaneous strength at 77°K. If σ_0 is independent of temperature, this asymptote will be 1.0. If, as is likely from the decrease of E and α with increasing temperature, σ_0 decreases with increasing temperature,† then the asymptote will lie between about 0.8 (the largest observed value in Fig. 6) and 1.0. The fact that all the room-temperature fatigue data of this study can be fitted to the same curve when plotted in terms of σ_N indicates that σ_0/σ_N equals a constant for different values of σ_N and one can write $\sigma_0 = k\sigma_N$. The reduced strength, σ/σ_N, of Fig. 5 is then equal to $k\sigma/\sigma_0$, the strength relative to the instantaneous strength at room temperature. This relation is needed in comparing the experimental data with various theories.

(2) *Comparison of UFC with Various Theories*

Several equations for the static fatigue of glass which have been derived by various investigators on the basis of different physical mechanisms are shown in Table VIII. To provide a ready basis for comparison they all have been written in a form which gives $\log t$, the time until breakage occurs, as a function of σ, the applied stress. In each case all the constants occurring in the theory can be combined into two positive parameters a and b, which are constant for any given specimens and set of test conditions. The constants a and b are not of course the same for the different expressions.

Expressions (*B*) and (*D*) in this table are asymptotic forms of more complete expressions derived by the respective investigators and should be valid for large and small values of σ respectively. To fit the complete fatigue curve their complete expressions are necessary.

An attempt was made to fit the universal fatigue curve of Fig. 6 with functions of the forms shown in Table VIII. The resulting curves are shown in Fig. 8 along with points (see Appendix I) representing the smoothed curve of Fig. 6. For each expression the curve of best fit was found by plotting the smoothed points of Fig. 6 in terms of variables such that the equation in question would yield a straight line. The straight line which gave the best fit over the expected region of validity of the equation was then used as the best theoretical expression. All these expressions were then replotted in the usual coordinates in Fig. 8.

From the figure it can be seen that expression (*A*) reproduces the data well for large $t/t_{0.5}$ but deviates badly for short times and predicts infinite strength for finite times within the experimentally accessible region. This expression was originally derived to fit the data of Baker and Preston.[15] Since their data were acquired for specimens with relatively high strength (see Fig. 3) and correspondingly small values of $t_{0.5}$ (since $\ln t_{0.5} \propto 1/\sigma_N^2$), their experimental points undoubtedly correspond to $t/t_{0.5}$ large. Thus expression (*A*) gives ex-

* The test duration is assumed to be long compared with the time (a few microseconds) required for a crack to travel through the specimen at the limiting crack velocity.

[25] R. H. Kropschot and R. P. Mikesell, "Strength and Fatigue of Glass at Very Low Temperatures," *J. Appl. Phys.*, 28, 610–14 (1957).

† By the Griffith relation; see preceding sections of this paper.

cellent agreement with Baker and Preston's data,[20] even though it does not apply for the small values of reduced time in the present results.

Expression (*B*) gives an excellent representation of the data over the range for which it is valid. In addition, from the direction of the deviation, it seems likely that Stuart and Anderson's complete curve, of which this is an asymptotic form, would also fit the rest of the data for longer times. It should be noted, however, that fitting the long-time end of the curve involves, in this case, introducing an additional constant into the expression so that it becomes a three-parameter, rather than a two-parameter, theory. In addition this expression is inconsistent with the existence of an instantaneous strength since as t becomes indefinitely small, σ increases indefinitely.

Expression (*C*) did not provide a good fit to the data over any of the experimental range for intervals of more than three or four decades. The curve shown gives the best fit for large load durations. A curve which fits the short-time data would yield strength values much too high at long times. This expression, like (*A*), predicts infinite strength for some short but measurable value of load duration.

Finally, expression (*D*) reproduced the data very well over the expected region of validity. In this case the complete expression, which has a horizontal asymptote for short times, should also give a reasonable fit to the experimental curve over its entire range. Charles' expression therefore reproduces all the features of the UFC as determined in this study.

The foregoing analysis has shown that expressions of the general form of (*A*) and (*C*) must be considered as approximations valid only over a very limited region of the complete static fatigue curve. Expressions (*B*) and (*D*) fit the data equally well over the regions for which they are valid, and the complete expressions of the respective theories would fit the entire experimental curve with reasonable accuracy. However, (*B*) predicts a value of σ increasing indefinitely with decreasing load duration beyond the range of the experimental data. Thus among the four theories considered, only that of Charles, (*D*), yields a time dependence in complete agreement with the data obtained in this study.

(3) Dependence of $t_{0.5}$ on $1/\sigma_N^2$

In order to compare the various theories with the second experimental criterion ((*b*) above), note that the constants a and b in expressions (*A*) through (*D*) of Table VIII are constant only with respect to a given set of specimens and experimental conditions. They can in general depend on σ_0, the initial strength of the specimens, and also on the variables (e.g. temperature, relative humidity in air, and pH in liquid) which characterize the test conditions. As mentioned above, the dependence on such variables may provide a much more critical test than the degree of agreement with the UFC.

One can write equations (*A*) through (*D*) in terms of σ/σ_N as follows:

$$(A')\ \log t = -a + \frac{b}{\sigma_N}\left(\frac{\sigma_N}{\sigma}\right) \tag{5}$$

$$(B')\ \log t \simeq a - \log \sigma_N - b\sigma_N\left(\frac{\sigma}{\sigma_N}\right) - \log\left(\frac{\sigma}{\sigma_N}\right) \tag{6}$$

$$(C')\ \log t = -a + \frac{b}{\sigma_N^2}\left(\frac{\sigma_N}{\sigma}\right)^2 \tag{7}$$

$$(D')\ \log t \simeq -a - b\log \sigma_N - b\log\left(\frac{\sigma}{\sigma_N}\right) \tag{8}$$

If $\sigma/\sigma_N = 0.5$ is substituted in these equations, the expressions for $\log t_{0.5}$ are in terms of a, b, and σ_N. Furthermore, theories (*B*) and (*D*) were developed in such a way that for these theories it is possible to write the explicit dependence of a and b on the initial crack depth σ_0 and thus on $1/\sigma_0^2 = 1/k^2\sigma_N^2$.

In the case of (*B*) (Stuart and Anderson[22]) the theory contains a stress-concentration factor, d (c in their notation), which by the theory of stress concentrators can be expressed in terms of the initial crack depth, c_0, or low-temperature strength, σ_N, as follows (noting that $\sigma_0 = k\sigma_N$):

$$d \simeq 2\sqrt{c_0} = \frac{K}{\sigma_0^2} = \frac{K}{k^2\sigma_N^2} \tag{9}$$

In the case of (*D*) (Charles[24]), the initial crack depth occurs explicitly in the development of the theory.

For the various developments of theory (*A*) neither stress concentrators nor instantaneous strength play any role so there is no attempt to apply it here. For theory (*C*) the original form as stated by Elliott[23] implies that the crack depth is negative and infinite for $t = 0$ and thus cannot be applied. A simple modification, however, eliminates this difficulty without affecting the long-time behavior of the theory (see Appendix II, p. 592). With this modification one can also predict a dependence of $t_{0.5}$ on $1/\sigma_N^2$ from this theory.

The predicted dependences of $t_{0.5}$ on $1/\sigma_N^2$ for theories (*B*), (*C*), and (*D*) as obtained in this way are given by

$$(B)\ \log t_{0.5} = \text{constant (independent of } \sigma_N) \tag{10}$$

$$(C)\ \log t_{0.5} = A + \frac{B}{\sigma_N^2} \tag{11}$$

$$(D)\ t_{0.5} = \frac{A}{\sigma_N^2} \tag{12}$$

where A and B are independent of σ_N. Of the three treatments, only (*C*) (Elliott, modified as in Appendix II) yields a dependence in agreement with the results of this study as shown in Fig. 6 and expressed in equation (3).

(4) General Conclusion

None of the static fatigue theories considered above provides a complete and adequate account of all the results of this study. Two of the approaches, however (Charles[24] and Elliott,[23] modified), are partly successful, each being in agreement with one of the experimental criteria. By examining the bases of these approaches one may obtain clues as to the probable nature of a more complete theory.

In each of these approaches static fatigue is assumed to depend on the growth of microscopic cracks in the surface of a glass specimen under stress until they reach such a size that the instantaneous strength is equal to the applied stress. The treatments differ with respect to the assumed dependence of the rate of crack growth (dc/dt) on other variables.

Charles assumes that dc/dt is an explicit function only of the instantaneous value of σ_t, the localized stress at the crack tip. Any dependence on c, the instantaneous crack depth, arises only through the effect of c on the stress-concentration factor for the crack and thus on σ_t. As seen above, such a treatment can provide the proper dependence of the break time on applied stress but does not yield the correct dependence of $t_{0.5}$ on c_0 or σ_0, the initial values of crack depth and instantaneous strength.

In Elliott's treatment as modified in Appendix II, dc/dt is assumed to depend explicitly on c but to be independent of σ. (It is, of course, implied that some tensile stress must be present for crack growth to occur.) This treatment yields the proper dependence of $t_{0.5}$ on c_0 and σ_0 but is not in agreement with the observed dependence of break time on applied stress.

If the crack-growth hypothesis is correct, it seems likely that a complete theory for the static fatigue of glass must contain features of both of the foregoing treatments. Specifically, the rate of crack growth may depend explicitly on the localized stress at the crack tip (as determined by σ, the applied macroscopic stress, and c, the crack depth) and also independently on c itself. This suggests that the rate of crack growth is determined by a stress-dependent interaction between glass and an attacking medium (in these experiments water) and that this interaction is also governed by the availability of the medium at the tip of the crack. Treatments which consider only the dependence on the stress (Charles) or

the availability of the medium (Elliott) yield only partial agreement with the experimental results. It seems likely that a complete theory must take account of both these factors.

Finally, it should be recalled that the experimental static fatigue results of the present paper were obtained with the specimens immersed in distilled water, and, strictly speaking, the foregoing theoretical discussion applies to this situation only. Strength and static fatigue in other ambient media will be treated in Part IV of this series.

IV. Summary and Conclusions

The results of this study can be summarized as follows:

(1) A controlled grit blast has been found to be a highly reproducible method of producing standardized damage to a glass surface. The effects of grit size, blast pressure, and amount of grit have been studied and described.

(2) For abrasions of simple geometry (emery scratches at right angles to the stress) the strength as measured with the specimens immersed in liquid nitrogen follows the Griffith relation $\sigma \approx \sqrt{E\alpha/c}$ if a value of α (the surface energy) considerably larger than that measured at high temperatures is used.

(3) Abrasions of different geometry and severity produce static fatigue curves of the same general nature but differing in quantitative detail for tests in distilled water. However, when normalized with respect to the very low temperature strength (which is independent of time) and plotted vs. a reduced time coordinate, all the curves can be fitted by a single universal fatigue curve.

(4) The breaking time ($t_{0.5}$) for stresses equal to half the low-temperature strength depends strongly on the geometry and severity of the flaws in the surface. Large flaws (low initial strength) show much less rapid weakening relative to their low-temperature strength than do small flaws. The rate of weakening under stress appears to depend exponentially on initial flaw size ($\ln t_{0.5} \propto 1/\sigma_N^2$).

(5) Linear flaws weaken more rapidly under stress than do point flaws by a factor of approximately 50.

(6) Of several published theories for the static fatigue of glass none gives an adequate account of the complete experimental results of this study. An examination of these theories in the light of the present data indicates the probable nature of a more complete theory.

Appendix I

Numerical Values for Universal Fatigue Curve

The numerical values of the UFC as obtained from the data of this study (Fig. 6) are shown in Table AI.

Table AI

$\frac{t}{t_{0.5}}$	$\frac{\sigma}{\sigma_N} = f\left(\frac{t}{t_{0.5}}\right)$
10^{-4}	0.82
10^{-3}	0.74
10^{-2}	0.66
10^{-1}	0.58
1	0.50
10	0.42
10^{2}	0.35
10^{3}	0.29
10^{4}	0.24
10^{5}	0.21

Appendix II

Modification of the Elliott Relation

Elliott[23] suggests that the growth of a surface crack under stress is governed by the diffusion of a corrosive agent through the amorphous corrosion product remaining in the crack. By analogy with the growth law for oxide films he writes:

$$c = \alpha \log t + \beta \tag{A1}$$

where c = crack depth at time t and α and β are constants. Applying the Griffith criterion:

$$\frac{1}{\sigma} = Kc^{1/2} \tag{A2}$$

he obtains

$$\frac{1}{\sigma^2} = \alpha_0 \log t + \beta_0 \tag{A3}$$

where σ = applied stress, t = time, and α_0 and β_0 are constants. For $t = 0$ equation (A1) gives $c = -\infty$ so that it must cease to apply for very small values of t. One can remove this difficulty by writing

$$\begin{aligned} c &= \alpha \log (t + t_0) + \beta \\ &= \alpha \log \left(\frac{t}{t_0} + 1\right) + \beta' \end{aligned} \tag{A4}$$

where

$$\beta' = \beta + \alpha \log t_0$$

Static fatigue involves a range of t covering many decades so that, except for the shortest durations, $t >> t_0$ and equation (A4) is equivalent to equation (A1). With this modification equation (A3) is replaced by

$$\frac{1}{\sigma^2} = \alpha_0 \log \left(\frac{t}{t_0} + 1\right) + \beta_0' \tag{A5}$$

for short times. For $t = 0$ (instantaneous strength) this yields

$$\frac{1}{\sigma_0^2} = K^2 c_0 = \beta_0' \tag{A6}$$

For longer times ($t >> t_0$) equation (A5) can now be written

$$\frac{1}{\sigma^2} \simeq \alpha_0 \log t - \alpha_0 \log t_0 + \frac{1}{\sigma_0^2} \tag{A7}$$

or (noting that $\sigma_0 = k\sigma_N$),

$$\log t = \left(\log t_0 - \frac{1}{\sigma_0 k^2 \sigma_N^2}\right) + \frac{1}{\alpha_0 \sigma_N^2}\left(\frac{\sigma_N}{\sigma}\right)^2 \tag{A8}$$

Substituting $\frac{\sigma}{\sigma_N} = 0.5$ and $t = t_{0.5}$ one obtains

$$\log t_{0.5} = \log t_0 + \frac{1}{\alpha_0}\left(4 - \frac{1}{k^2}\right)\frac{1}{\sigma_N^2} \tag{A9}$$

This is the same as equation (11) of the text if

$$A = \log t_0 \tag{A10}$$

and

$$B = \left(4 - \frac{1}{k^2}\right)\frac{1}{\alpha_0}$$